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Scaling exponents of the second-order structure function of turbulence

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Abstract. The Kolmogorov equation, which is an exact relationship between the second-order structure function $D_{LL}(r)$ and the third-order structure function $D_{LLL}(r)$, is applied to study the relative scaling of $D_{LL}(r)$ against $-D_{LLL}(r)$ according to the extended self-similarity (ESS) method. It is found that the relative ESS scaling exponent S_2 is greater than the real (or theoretical) inertial range scaling exponent ζ_2 of $D_{LL}(r)$ in the case of normal scaling ($\zeta_2 = 2/3$) as well as anomalous scaling ($\zeta_2 = 0.7$): $S_2 \geqslant 0.7$ when $\zeta_2 = 2/3$ and $S_2 > 0.72$ when $\zeta_2 = 0.7$. Therefore, the experimental and numerical results $S_2 = 0.7$ favours the Kolmogorov $2/3$ law ($\zeta_2 = 2/3$) rather than anomalous scaling ($\zeta_2 = 0.7$). Previously the results $S_2 = 0.7$ were interpreted as clear evidence of anomalous scaling ($\zeta_2 = 0.7$) based upon the assumption that $\zeta_2 = S_2$.

1. Introduction

The structure function of turbulence of order n is $\langle \Delta u_r^n \rangle$ or $\langle |\Delta u_r|^n \rangle$, where $\langle \cdots \rangle$ is the statistical average and Δu_r is the longitudinal velocity difference across a distance r. In the inertial range [1], we have the scaling $\langle \Delta u_r^n \rangle \sim r^{\zeta n}$, where ζ_n is the scaling exponent of order n. Strictly speaking, ζ_n is defined for the idealized model of inertial range corresponding to the asymptotic case of infinite Reynolds number [2], and is called the real (or theoretical) inertial range scaling exponent in this paper. By the Kolmogorov $2/3$ law [1], $\zeta_2 = 2/3$, the second-order structure function $D_{LL}(r)$ scales as $r^{2/3}$ in the inertial range; but various intermittency models predict that $\zeta_2 = 0.7$ [1]. It is very difficult to take accurate measurements to confirm whether $\zeta_2 = 2/3$ (normal scaling) or $\zeta_2 = 0.7$ (anomalous scaling). Recently, Benzi *et al* [3] proposed the extended self-similarity (ESS) method to improve the accuracy of the experimental scaling exponents, in which they make a log–log plot of the structure functions against the third-order structure function, and observe an extended scaling range up to about $r/\eta = 4$ in the dissipation range, which is much wider than that observed in the traditional plot against r; here $\eta = (\nu^3/\varepsilon)^{1/4}$ is the Kolmogorov scale. It is believed that the ESS method can determine accurately the scaling exponent experimentally or numerically. The scaling exponent derived by the ESS method is denoted by S_n and is called the ESS scaling exponent in this paper. Stolovitzky and Sreenivasan [4] pointed out that the concept of ESS is valid for low-order structure functions, but is questionable for high-order structure functions. In the case of $n = 2$, by using experimental data at $R_\lambda = 10^2$–10^3, Benzi *et al* [3] obtain $S_2 = 0.7$ over the ESS range $4 < r/\eta < 10^3$. By assuming $\zeta_2 = S_2$, they reported that the result $S_2 = 0.7$ is clear evidence of anomalous scaling ($\zeta_2 = 0.7$) of $D_{LL}(r)$. Several different independent experiments give the same result that $S_2 = 0.7$ (see figure 3 of [5]). Cao *et al* [6] use direct numerical simulation (DNS) of Navier–Stokes turbulence to determine the ESS scaling exponent S_n of low-order structure functions, and obtain $S_2 = 0.7$ in agreement with experiments. By assuming $S_2 = \zeta_2$, they conclude that the DNS result means unambiguously the anomalous scaling ($\zeta_2 = 0.7$) of $D_{\mathrm{LL}}(r)$.

In this paper, we make a thorough theoretical analysis of the relative scaling of $D_{\mathrm{LL}}(r)$ versus $-D_{\mathrm{LLL}}(r)$, where $D_{\mathrm{LLL}}(r) = \langle \Delta u_r^3 \rangle$ is the third-order structure function. Our results are:

(a) in the log–log plot of $D_{\mathrm{LL}}(r)$ against $-D_{\mathrm{LLL}}(r)$, an approximate relative scaling law is observed over the range $4 < r/\eta < 10^3$, which is the ESS range adopted by Benzi *et al* while they obtain $S_2 = 0.7$;

(b) the local slope of the log–log plot is not constant and has a remarkable bump in the ESS range, so that the relative scaling law is not valid in a strict sense;

(c) the ESS scaling exponent S_2 is greater than the real inertial range scaling exponent ζ_2 for both normal scaling ($\zeta_2 = 2/3$) and anomalous scaling ($\zeta_2 = 0.7$), i.e. $S_2 \geqslant 0.7$ for $\zeta_2 = 2/3$, and $S_2 > 0.72$ for $\zeta_2 = 0.7$.

The experimental and numerical results $S_2 = 0.7$ have previously been interpreted as clear evidence of anomalous scaling ($\zeta_2 = 0.7$) of $D_{\mathrm{LL}}(r)$ based upon the assumption that $\zeta_2 = S_2$. In contrast, our results show that the data $S_2 = 0.7$ actually favour the Kolmogorov 2/3 law ($\zeta_2 = 2/3$) rather than anomalous scaling ($\zeta_2 = 0.7$).

2. Outline of methods

In the universal equilibrium range, we have the following exact relationship between the second- and third-order structure functions [1]

$$D_{\mathrm{LLL}}(r) = -(4/5)\varepsilon r + 6\nu \, \mathrm{d}D_{\mathrm{LL}}(r)/\,\mathrm{d}r \tag{1}$$

which is the celebrated Kolmogorov equation. Here ε is the energy dissipation rate, and ν is the kinematic viscosity. When the second-order structure function $D_{\mathrm{LL}}(r)$ is known, the third-order structure function $D_{\mathrm{LLL}}(r)$ is obtained from (1), and then we can calculate the ESS scaling exponent S_2 of $D_{\mathrm{LL}}(r)$ by the ESS method.

$D_{\mathrm{LL}}(r)$ is related to the three-dimensional (3D) and one-dimensional (1D) energy spectrum $E(k)$ and $E_1(k)$ (see Monin and Yaglom [1])

$$D_{\mathrm{LL}}(r) = 4\int_0^\infty E(k)[1/3 + \cos(kr)/(kr)^2 - \sin(kr)/(kr)^3]\,\mathrm{d}k \tag{2}$$

$$D_{\mathrm{LL}}(r) = 2\int_0^\infty E_1(k)[1 - \cos(kr)]\,\mathrm{d}k. \tag{3}$$

The longitudinal structure function $D_{\mathrm{LL}}(r)$ is related to the isotropic structure function $D(r)$ [1, 7]

$$D(r) = 3D_{\mathrm{LL}}(r) + r\,\mathrm{d}D_{\mathrm{LL}}(r)/\,\mathrm{d}r \tag{4a}$$

and

$$D_{\mathrm{LL}}(r) = r^{-3}\int_0^r x^2 D(x)\,\mathrm{d}x. \tag{4b}$$

Although an exact expression for $D_{\mathrm{LL}}(r)$ is not available at present, there are various approximate models for $E(k)$, $E_1(k)$ and $D(r)$, based upon experimental, numerical and theoretical works of more than half a century. By using these models and equations (2)–(4) we can calculate $D_{\mathrm{LL}}(r)$, and then calculate $D_{\mathrm{LLL}}(r)$ by using (1). In the log–log plot of

$D_{LL}(r)$ against $-D_{LLL}(r)$ we observe an approximate relative scaling law over the ESS range adopted by Benzi *et al* while they obtain $S_2 = 0.7$. However, the local slope of the log–log plot is not constant and has a remarkable bump in the ESS range; hence, the relative scaling law is not valid in a strict sense. Finally, we calculate the ESS scaling exponent S_2 of $D_{LL}(r)$ by the ESS method for both normal scaling ($\zeta_2 = 2/3$) and anomalous scaling ($\zeta_2 = 0.7$).

3. Proof for typical models of the energy spectrum

First, we study the case of normal scaling ($\zeta_2 = 2/3$). In the universal equilibrium range, we have [1]

$$E(k) = Ko\varepsilon^{2/3}k^{-5/3}F(k/k_d) \qquad F(0) = 1 \tag{5}$$

where Ko is the Kolmogorov constant and $F(x)$ is a universal function of $x = k/k_d$. From (5) and the energy dissipation relationship, we obtain

$$2Ko\int_0^\infty x^{1/3}F(x)\,\mathrm{d}x = 1. \tag{6}$$

A simple model of $F(x)$ [1] is

$$F(x) = \exp(-Cx^n). \tag{7}$$

There are three undetermined parameters Ko, C and n in equations (5) and (7), but only two of them are independent due to the constraint (6). Many efforts [1, 8] have been made to estimate the value of n: Kraichnan proposed $n = 1$, Pao suggested $n = 4/3$, and the theoretical analysis by Foias *et al* shows that $n = 1$ for the far dissipation range. As $n \to \infty$, (7) with (6) become the instructive discrete model

$$F(x) = 1 \qquad \text{if } x < x_c \qquad \text{and} \qquad F(x) = 0 \qquad \text{if } x > x_c \tag{8}$$

where $x_c = (1.5Ko)^{-0.75}$. When n and Ko are given and C in (7) is determined by (6), then we can calculate $D_{LL}(r)$ by using (2), (5) and (7), and finally calculate $D_{LLL}(r)$ by (1). In a log–log plot of $D_{LL}(r)$ against $-D_{LLL}(r)$, a straight line fitting the result over the ESS range is found by the least-squares method, and its slope is the ESS scaling exponent S_2 of $D_{LL}(r)$. The ESS range is $4 < r/\eta < 10^3$, which is the same as that adopted by Benzi *et al* [3] while they obtain $S_2 = 0.7$. For illustration, figure 1 shows the log–log plot of $D_{LL}(r)$ against $-D_{LLL}(r)$ over the ESS range for Pao's formula, and figure 2 shows its local slope

$$S_L = \mathrm{d}\log(D_{LL}(r))/\mathrm{d}\log(-D_{LLL}(r))$$

over the range $0.1 < r/\eta < 10^6$. Pao's formula [8] is a special case of the model (7) when $n = 4/3$ and $Ko = 1.7$, and from (6) we obtain $C = 2.55$. It is clear from figure 1 that we have an approximate relative scaling law over the ESS range. However, the local slope S_L shown in figure 2 is not constant and has a remarkable bump in the ESS range $4 < r/\eta < 10^3$; hence, the relative scaling law is not valid in a strict sense. Figure 3 gives the ESS scaling exponent S_2 of the model (7) for $2 \geqslant Ko \geqslant 1$ and $\infty \geqslant n \geqslant 1$. The Kolmogorov constant Ko estimated by various methods [9, 10] scatters and is between 1 and 2. Figure 3 clearly shows that the ESS scaling exponent S_2 is greater than the real inertial range scaling exponent $\zeta_2 = 2/3$ no matter what the parameters of the model (7) are.

One might argue that S_2 being greater than ζ_2 shown in figure 3 is just a disadvantage of the simple model (7). The main drawback of this model (7) is that it neglects the bump phenomenon. In the following we study several typical models which take into account the

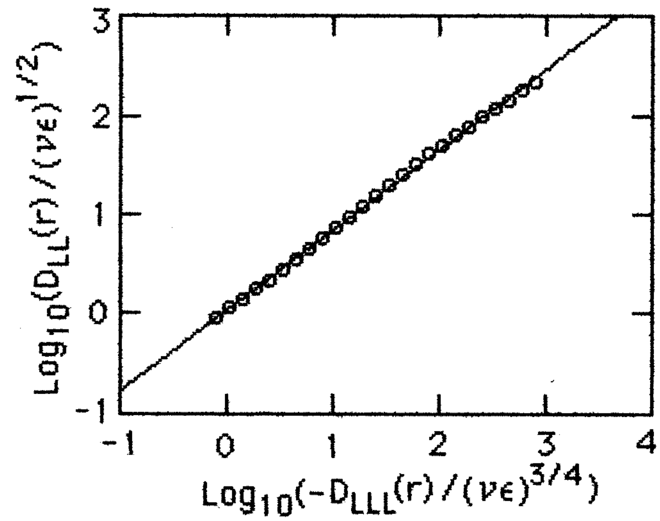

Figure 1. $D_{\mathrm{LL}}(r)$ against $-D_{\mathrm{LLL}}(r)$ over the ESS range $4 < r/\eta < 10^3$ for Pao's formula, $Ko = 1.7$, $F(x) = \exp(-2.55x^{4/3})$ (open circles); least-squares fit (full curve).

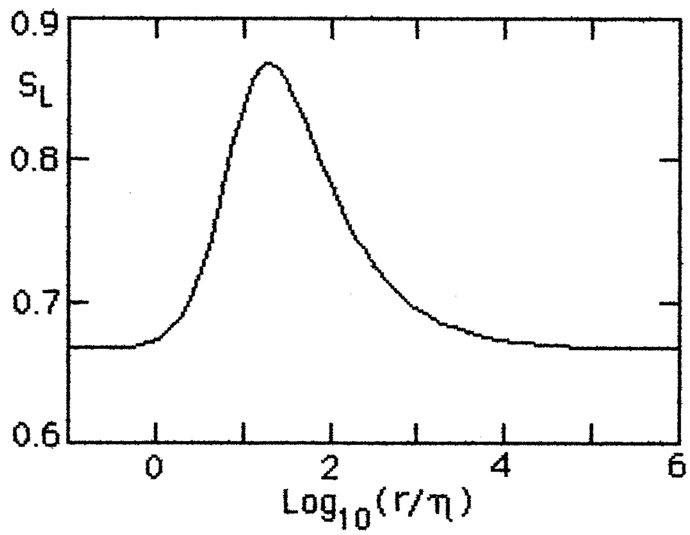

Figure 2. $S_{\mathrm{L}} = \mathrm{d}\log(D_{\mathrm{LL}}(r))/\mathrm{d}\log(-D_{\mathrm{LLL}}(r))$ against r/η for Pao's formula.

bump phenomenon, and show that their ESS scaling exponents are also greater than the real inertial range scaling exponent ζ_2. The first bump model is the one proposed in [11]

$$F(x) = (1 + Bx^m)\exp(-Cx^n) \qquad B > 0. \tag{9}$$

If $B = 0$, (9) becomes (7). According to (7), $F(x)$ is a monotonically decreasing function of x. In contrast, according to the bump model (9) where $B > 0$, $F(x)$ is not a monotonically decreasing function of x, but has a bump between the inertial and dissipation ranges. As x increases from zero to infinity, first $F(x)$ increases from $F(0) = 1$ to its maximum value at the bump centre and then decreases exponentially to zero. Numerical simulations of

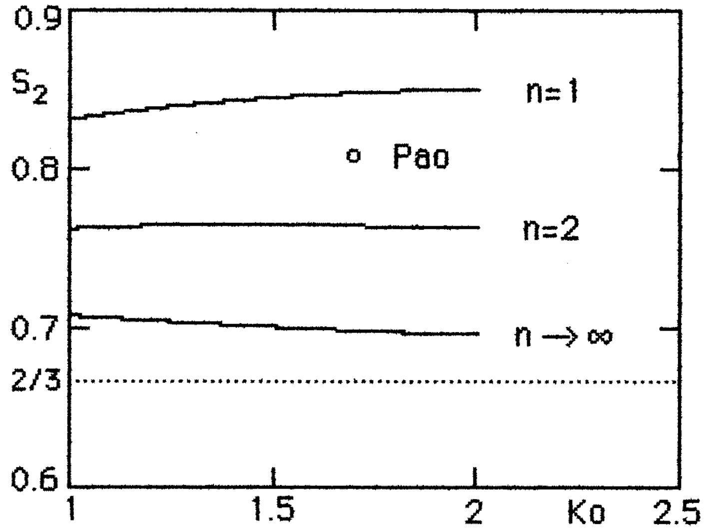

Figure 3. The ESS scaling exponent S_2 against Ko of the simple model equation (7) for $n = 1$, 2 and ∞ (full curve); Pao's formula (open circles).

isotropic turbulence and experiments [12, 13] confirm the existence of the bump. We have four adjustable parameters B, m, C and n in the bump model (9), but only three of them are independent due to the constraint (6) while Ko is given. The following two typical cases have been studied in [11]

$$Ko = 1.2 \qquad m = 2/3 \qquad C = 5.4 \qquad n = 4/3 \tag{10a}$$

$$Ko = 1.2 \qquad m = 1 \qquad C = 6.1 \qquad n = 1 \tag{10b}$$

and B is determined by the constraint (6). As shown in Qian and Gao [11], equations (10a) and (10b) give nearly the same 1D energy spectrum in the range $k/k_d < 0.5$ (or $r/\eta > 2$), and are in agreement with the experimental 1D spectrum. Similarly to (7), the ESS scaling exponent S_2 of the bump model (9) can be calculated by using equations (1), (2) and (5). The ESS scaling exponent S_2 of the bump model (9) is given in figure 4 for $(m, n) = (2/3, 4/3)$ and figure 5 for $(m, n) = (1, 1)$, and is also greater than the real inertial range scaling exponent $\zeta_2 = 2/3$. S_2 increases as Ko increases. Other choices of (m, n) give similar results. Experiments and numerical simulations [13] show that the parameter C is between 5 and 9 while $n = 1$.

The second bump model is for the 1D energy spectrum $E_1(k)$. According to the experimental 1D spectra of R_λ ranging from 130–13 000, She and Jackson proposed the model [14]

$$E_1(k)/E_1(k_p) = (k/k_p)^{-5/3} f_1(k/k_p) \tag{11a}$$

$$f_1(x)/f_1(0) = (1 + \beta x^{2/3}) \exp(-\mu x) \qquad \beta = 0.8. \tag{11b}$$

Here k_p is the maximum dissipation wavenumber, and μ is determined by the requirement that $k^2 E_1(k)$ attains its maximum at $k = k_p$. The She–Jackson model (11) is a special case of the 1D bump model

$$E_1(k) = (18/55) Ko \varepsilon^{2/3} k^{-5/3} F_1(k/k_p) \tag{12a}$$

$$F_1(x) = (1 + B_1 x^m) \exp(-C_1 x^n). \tag{12b}$$

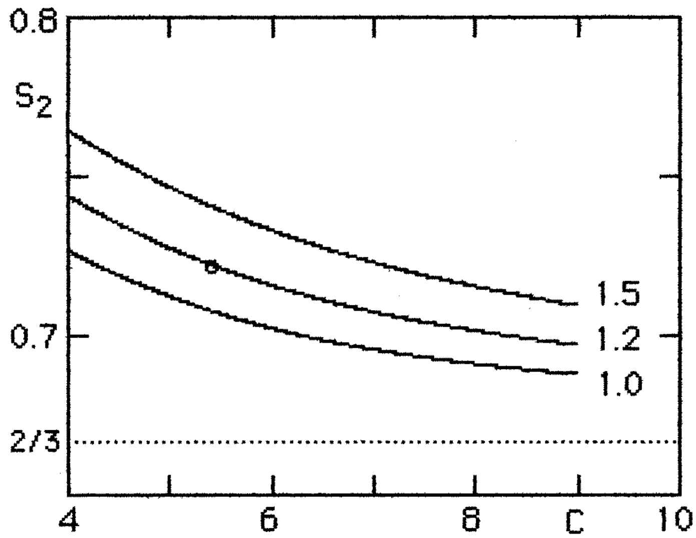

Figure 4. The ESS scaling exponent S_2 against C of the 3D bump model equation (9) for $Ko = 1.5$, 1.2 and 1.0 while $m = 2/3$ and $n = 4/3$ (full curve); equation (10*a*) (open circles).

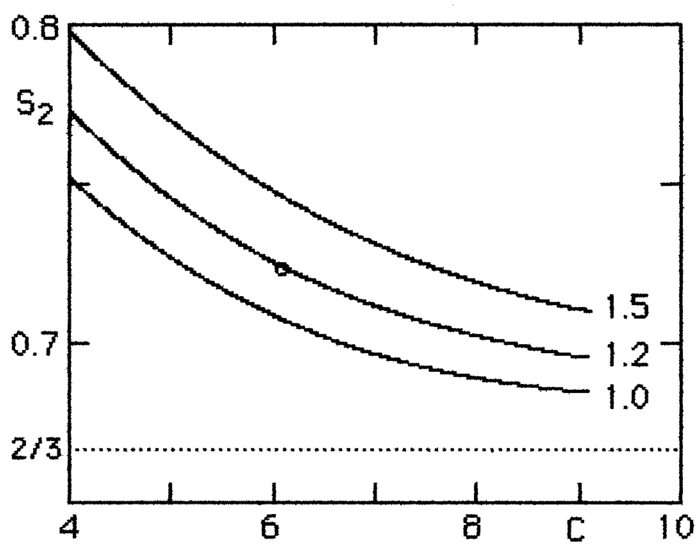

Figure 5. The ESS scaling exponent S_2 against C of the 3D bump model equation (9) for $Ko = 1.5$, 1.2 and 1.0 while $m = 1$ and $n = 1$ (full curve); equation (10*b*) (open circles).

From (12) and the energy dissipation relationship, we have

$$(54/11)Ko \int_0^\infty x^{1/3} F_1(x)\,\mathrm{d}x = 1. \qquad (12c)$$

When $m = 2/3$, $n = 1$, $B_1 = 0.8(k_d/k_p)^{2/3}$ and $C_1 = \mu(k_d/k_p)$, (12) becomes (11), and (12*c*) becomes (see Qian (1996) [10])

$$18.3Ko = (k_d/k_p)^{4/3}.$$

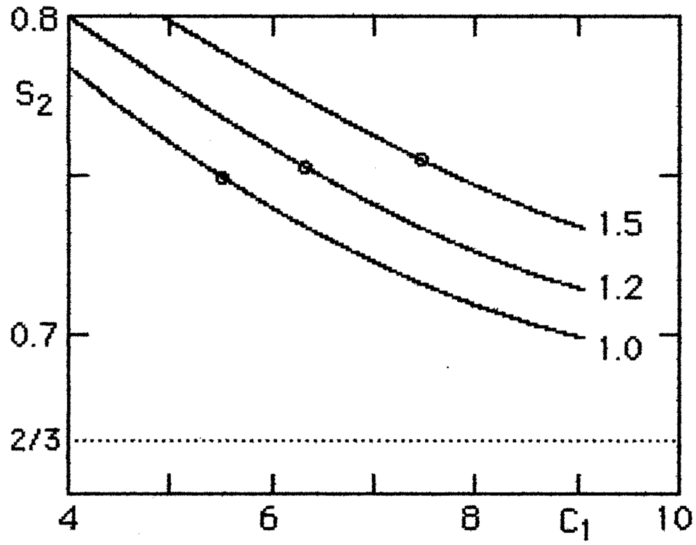

Figure 6. S_2 against C of the 1D bump model equation (12) for $Ko = 1.5$, 1.2 and 1.0 while $m = 2/3$ and $n = 1$ (full curve); She–Jackson formula equation (11) (open circles).

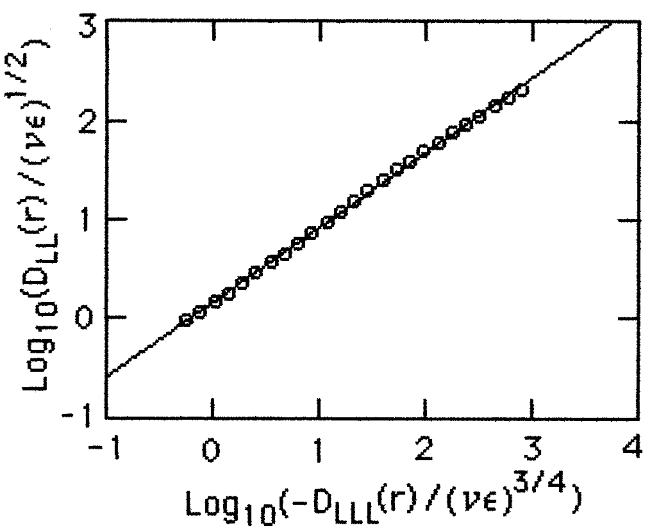

Figure 7. $D_{\mathrm{LL}}(r)$ against $-D_{\mathrm{LLL}}(r)$ over the ESS range $4 < r/\eta < 10^3$ for She–Jackson formula equation (11), $Ko = 1.5$ (open circles); least-squares fit (full curve).

The ESS scaling exponent S_2 of the 1D bump model, obtained by using (12), (3) and (1), is given in figure 6, and is also greater than the real inertial range scaling exponent $\zeta_2 = 2/3$. For illustration, the log–log plot of $D_{\mathrm{LL}}(r)$ against $-D_{\mathrm{LLL}}(r)$ over the ESS range for the She–Jackson model (11) is given in figure 7, and an approximate relative scaling law is observed over the ESS range $4 < r/\eta < 10^3$. However, its local slope S_{L} shown in figure 8 has a remarkable bump in the range.

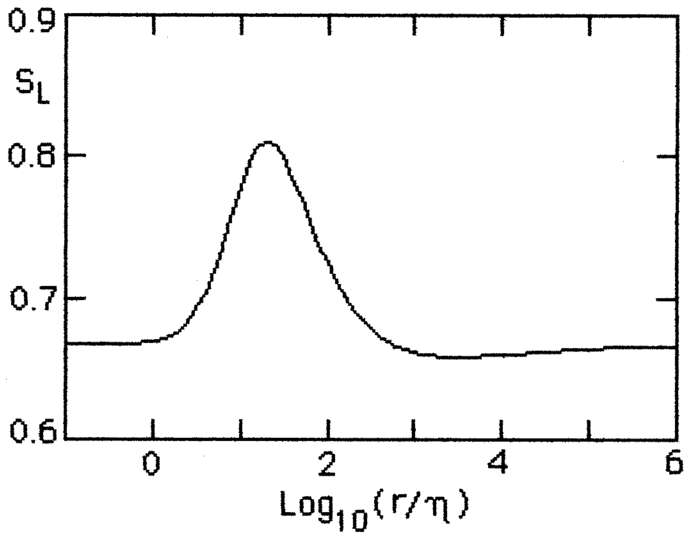

Figure 8. The local slope S_L against r/η of She–Jackson formula equation (11), $Ko = 1.5$.

The third bump model is that studied by Qian [2] and is

$$F(x) = (1 + Bx^{\alpha} g)\exp(-Cx^{\beta}) \tag{13a}$$

$$g = [1 + C_1 Z + C_2 Z^2 + \cdots + C_m Z^m]^2 \qquad Z = x^{\gamma} \tag{13b}$$

which is an improvement on the first bump model (9). Under the constraint (6), the parameters B, α, C, β, C_1, $C_2, \ldots, C_m$, and γ are adjusted to minimize the equation error of the spectral form of the von Karman–Howarth equation. The minimization is made for $m = 5$ in [2] for three typical values of Ko, and the equation error is less than 0.002. By equations (1), (2), (5) and (13), the result of [2] is adopted here to calculate the ESS scaling exponent S_2, which is given in table 1 and is also greater than the real inertial range scaling exponent $\zeta_2 = 2/3$. Figure 9 shows the log–log plot of $D_{LL}(r)$ against $-D_{LLL}(r)$ over the ESS range $4 < r/\eta < 10^3$ for the third bump model (13) while $Ko = 1.2$, and an approximate relative scaling law is clearly observed. Its local slope S_L is shown in figure 10 and has a remarkable bump in the ESS range, hence the relative scaling law is not valid in a strict sense.

Table 1. ESS scaling exponent S_2 and inertial range scaling exponent ζ_2.

	S_2			
Model	$Ko = 1.2$	$Ko = 1.5$	$Ko = 1.8$	ζ_2
(13)	0.710	0.728	0.739	2/3
(14)	0.707	0.708	0.709	2/3
(15)	0.725	0.725	0.724	0.7

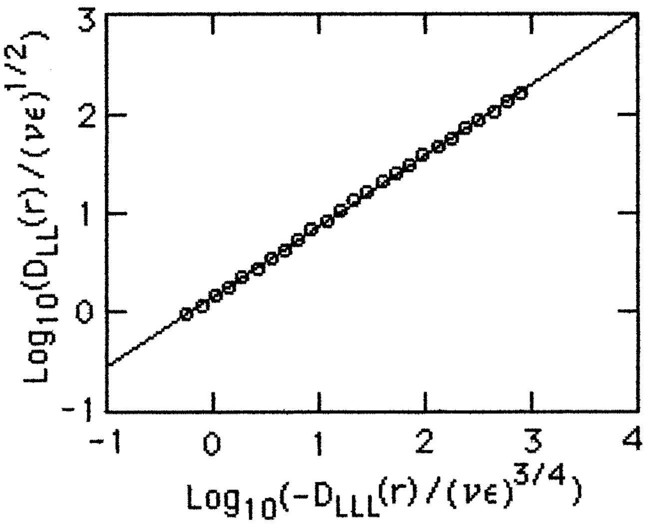

Figure 9. $D_{\mathrm{LL}}(r)$ against $-D_{\mathrm{LLL}}(r)$ over the ESS range $4 < r/\eta < 10^3$ for the third bump model equation (13), $Ko = 1.2$, and equation (10*a*) of [2] is used (open circles); least-squares fit (full curve).

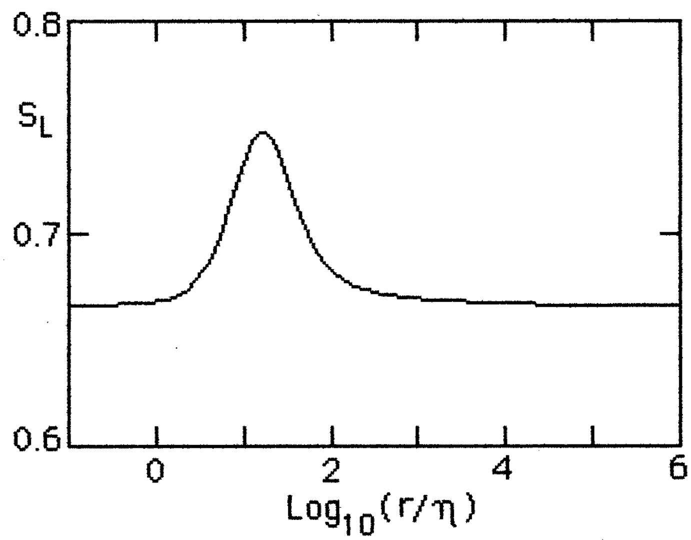

Figure 10. The local slope S_{L} against r/η of the third bump model equation (13), $Ko = 1.2$, and equation (10*a*) of [2] is used.

4. Proof for Batchelor's model of the structure function

At this point, one might argue that the conclusion from section 3 is valid in the case of normal scaling where $\zeta_2 = 2/3$, but might not be valid in the case of anomalous scaling. In this section, we show that the same conclusion is valid for the case of anomalous scaling where $\zeta_2 = 0.7$.

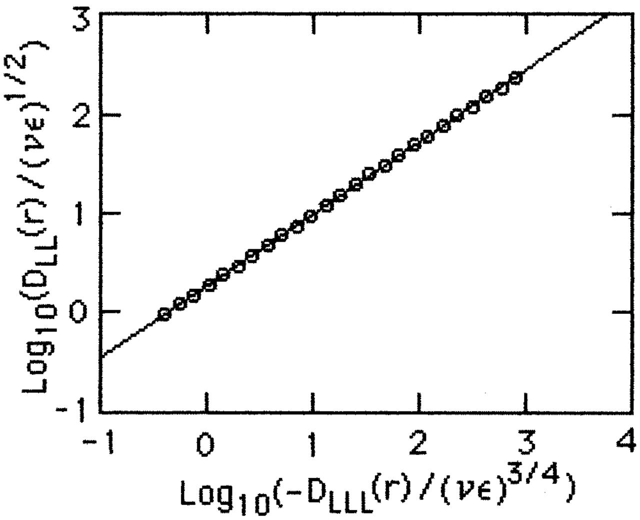

Figure 11. $D_{\rm LL}(r)$ against $-D_{\rm LLL}(r)$ of Batchelor's model equation (14) for the case of anomalous scaling $\zeta_2 = 0.7$, $Ko = 1.5$ (open circles); least-squares fit (full curve).

The experimental data of the second-order structure function can be fitted by the Batchelor interpolation formula [1], its quality is better in the isotropic case of $D(r)$ than in the longitudinal case of $D_{\rm LL}(r)$ [7]. The Batchelor interpolation formula for $D(r)$ is ([7] and references therein)

$$D(r)/(\nu\varepsilon)^{1/2} = (x^2/3)/[1 + (3C_k)^{-3/2}x^2]^{(1-\zeta_2/2)} \qquad x = r/\eta \tag{14a}$$

where C_k is a dimensional constant related to the Kolmogorov constant Ko

$$C_k = (9/5)\Gamma(1/3)Ko \tag{14b}$$

and $\Gamma(n)$ is the Gamma function. The longitudinal $D_{\rm LL}(r)$ is calculated by substituting (14) into (4), then $D_{\rm LLL}(r)$ is determined by the Kolmogorov equation (1) and finally the ESS scaling exponent S_2 is obtained by the ESS method. The ESS scaling exponents of Batchelor's model (14) for both anomalous scaling ($\zeta_2 = 0.7$) and normal scaling ($\zeta_2 = 2/3$) are given in table 1, and are all greater than the real inertial range scaling exponent ζ_2. Figure 11 shows the log–log plot of $D_{\rm LL}(r)$ against $-D_{\rm LLL}(r)$ of the Batchelor model (14) for the case of anomalous scaling, $\zeta_2 = 0.7$, when $Ko = 1.5$, and its local slope $S_{\rm L}$ is given in figure 12. As r/η increases, the local slope $S_{\rm L}$ reaches its maximum in the range $10 < r/\eta < 10^2$, and then has a slight minimum in the range $10^2 < r/\eta < 10^3$, finally approaching the inertial range scaling exponent ζ_2 when $r/\eta > 10^3$. An approximate relative scaling law is clearly observed over the ESS range in figure 11. However, the local slope $S_{\rm L}$ shown in figure 12 is not constant in the ESS range, so that the relative scaling law is not valid in a strict sense. The situation of normal scaling ($\zeta_2 = 2/3$) of (14) is similar to figures 9 and 10, and will not be shown here. The energy spectrum of the Batchelor model (14) has a bump between the inertial range and the dissipation range; therefore, from the point of view of the spectral dynamics, the Batchelor model (14) is also a bump model, the fourth bump model studied in this paper. When Batchelor's interpolation formula is used for $D_{\rm LL}(r)$ instead of $D(r)$, a similar result is obtained.

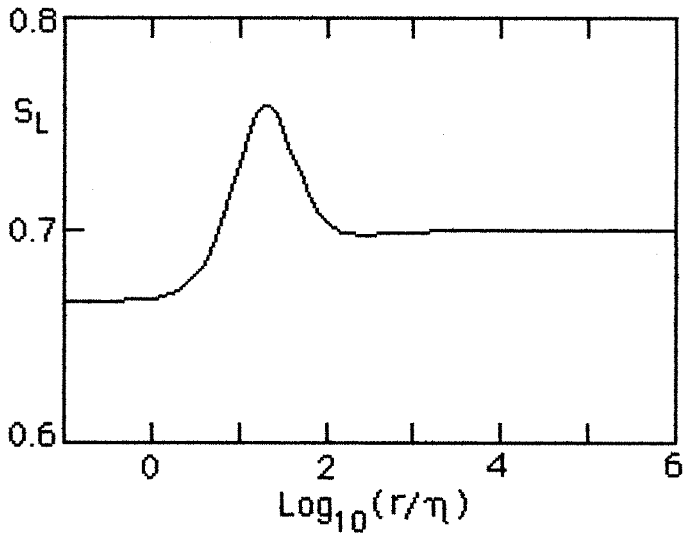

Figure 12. The local slope S_L against r/η of Batchelor's model equation (14) for the case of anomalous scaling $\zeta_2 = 0.7$, $Ko = 1.5$.

5. Discussion

By using the Kolmogorov equation (1) and five typical models of the second-order statistical moments (energy spectrum and the second-order structure function), we have studied the relative scaling of $D_{LL}(r)$ against $-D_{LLL}(r)$ following the ESS method. By adjusting the relevant parameters (for example, n, Ko, C and C_1) of these models in a wide range compatible with experimental data, these models can cover various possibilities. All models predict that the local slope S_L has a remarkable bump in the ESS range and that the ESS scaling exponent S_2 is greater than the real inertial range scaling exponent ζ_2, although different models predict different size and skirt of the bump.

In the log–log plot of $D_{LL}(r)$ against $-D_{LLL}(r)$, as shown in figures 1, 7, 9 and 11, an approximate relative scaling law is observed over the ESS range $4 < r/\eta < 10^3$ adopted by Benzi *et al* [3] while they obtain $S_2 = 0.7$. However, as shown in figures 2, 8, 10 and 12, the local slope S_L is not constant and has a remarkable bump in the range $4 < r/\eta < 10^3$, so the relative scaling law is not valid in a strict sense. As a consequence, the ESS scaling exponent S_2 is greater than the real inertial range scaling exponent ζ_2 for both normal scaling ($\zeta_2 = 2/3$) and anomalous scaling ($\zeta_2 = 0.7$), i.e. $S_2 \geqslant 0.7$ when $\zeta_2 = 2/3$ and $S_2 > 0.72$ when $\zeta_2 = 0.7$. The experimental and numerical results $S_2 = 0.7$ have previously been interpreted as clear evidence of anomalous scaling ($\zeta_2 = 0.7$) of $D_{LL}(r)$ based upon the assumption that $\zeta_2 = S_2$. In contrast, our results show that the data $S_2 = 0.7$ actually favour the Kolmogorov 2/3 law ($\zeta_2 = 2/3$) rather than anomalous scaling ($\zeta_2 = 0.7$).

Acknowledgments

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